23 Optimization for Continuous Shortest Paths in Transportation

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CONTENTS

23.1 INTRODUCTION AND MOTIVATION

The shortest path problem is a fundamental task in path-planning area and it has been the subject of extensive research resulting in publication of numerous scientific papers. Problem of planning shortest paths, in fact, is one of the most powerful tools for modeling combinatorial optimization problems. The constituents of a general problem for path planning are *environment* (e.g., a network, a graph, a tree, or a geometric domain); *constraints* (obstacle avoidance, short length, number of turns depending on the specific application); and a combination of *criteria* (e.g., optimizing distances or cost functions). Shortest paths in graphs and network (i.e., finding a path connecting two vertices with minimum total length) are well studied, and this classical optimization problem received a lot of attention with significant progress (Ahuja et al., 1983; Pallottino and Scutella, 1998). In the case that all edge weights are non-negative, the graph had no negative cycles, and the well-known Dijkstra's algorithm (Dijkstra, 1959) allowed computing a tree of shortest paths from any source node to all other nodes. This approach is a crucial tool both AQ1 for discrete and continuous environment. In fact, although the problems dealt with in this survey have a continuous formulation, these can be reduced to search in a discrete structure due to a finite solution set. Since it is a basic tool modified many times to solve different problems in this context, the Dijkstra's paradigm is briefly introduced in the next section.

The continuous version of the shortest paths problem has also been oriented to robotic applications. This problem in a geometric environment gives a different flavor where encoding of edges is non-explicit, and it is usually specified by moving geometric object. A typical motion planning problem can then be defined as follows: Given a d-dimensional space strewn with *"obstacles*," compute a path connecting two points such that the path does not intersect the interior of any obstacle to minimize the total length of the path (based on a certain metric). It is one of the most fundamental topics in computational geometry (Mitchell, 1997,2000), and the main problem in this scenario is computing the shortest path in a polygonal domain. Many methods proposed to solve this problem are based on visibility graphs and the Dijkstra's algorithm. The *visibility graph* of a polygonal region *P* is the one where nodes are the vertices of *P*, and the edges join pair of nodes where the connecting segment lies inside the region *P* (de Berg et al., 2000) (Figure 23.1). The edges can be weighed by some measure, for instance with their Euclidean distance. Once the visibility graph (Ghosh, 1991 presents an efficient construction) is computed, the Dijkstra's algorithm can be used to compute a tree of shortest paths from the source to all vertices in *P*. AQ2

Among the well-studied areas of continuous shortest path optimization, i.e., network transportation and geometric shortest paths, a couple of problems modeling different interaction types between the route and the population around exist. From a theoretical point of view, these problems belong to the facility location and location routing fields (Laporte, 1992; Mina et al., 1998). Given a set of demand points, goal of classical location problems is to find one or several paths to optimize one or several possible constrained objective functions. Objective functions usually depend on the interactions among demand points and new facilities. When new facilities cannot be represented as points but some kind of dimensional sets, *extensive facility*

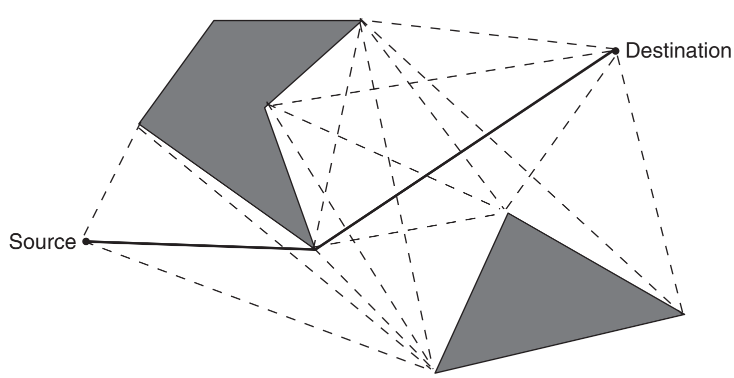

FIGURE 23.1 Visibility graph and the shortest source–destination path.

location problems arise. These problems consist of choosing an element in a class of (geometric) sets representing the candidate facilities that best fit the set of demand points according to specified criteria. In particular, location of straight lines, line segments, hyper-planes, spheres, and some types of polygonal curves have been studied for these problems (Díaz-Báñez et al., 2004).

The literature information for locating dimensional facilities is quite heterogeneous. One reason is that the results in this area are strongly dependent on the type of geometrical elements representing the facilities. For the location of straight lines or circles, good characterized results for optimal elements are available. However, due to the higher freedom degree of polygonal shaped facilities, not many results have been published for location of polygonal paths. Hence, in this area, the research studies aim to directly obtain efficient algorithms. Another reason for the heterogeneous information is due to that extensive facility location problems arise in different mathematical disciplines leading to different approaches used in proofs of the same result. Furthermore, the suggested algorithms show a wide variety since they are based on different techniques. Convex analysis, linear and dynamic programming and the most usual approaches of computational geometry can be given examples for different techniques.

This chapter focuses on continuous short paths problems, and an overview of the research for optimization approaches to the problem of optimally designing a transportation route in a continuous environment (continuous short path) is given. Typically, there are two important parameters in the optimal path design: the *cost of the path* (length or number of turns-vertices) and the *cost of the population* (related to the distance between path and population). The population cost can be further divided into two optimization criteria. On one hand, when a route for distributing goods to a set of customers represented by points in the plane is asked, the application of the *minmax criterion* (in which the distance from the path to the farthest point is minimized) is considered. On the contrary, when designing an *obnoxious route* (with potential undesirable effects, for instance, of a path for transportation of hazardous material), *the maxmin criterion* (in which the distance from the path to the closest point is maximized) is usually applied to locate the path as far as possible from the population. Note that although the *minsum criterion* (sum of the distances from all points to the path is minimized) is common in facility location theory (Drezner, 1995), it has not been considered to date for locating continuous polygonal paths. Wang (2002) and Tamir et al. (2005) give details for locating a path with *minsum* criteria in a tree network. Therefore, the objective of this chapter is first to provide a starting point for researchers and students interested in transportation field giving brief comments on existing literature and to present an overview of this optimization area for experienced researchers.

23.2 BASIC TERMINOLOGY AND DEFINITIONS

Formulation of the main problem considered in this chapter is: Given a source point s, a destination point d, a set $S = \{p_1, p_2, \dots p_n\}$ of n demand sites representing the population (points in a geometric domain), a path P (generally a polygonal path) connecting s and d such that a certain objective function is optimized subject to some constrains on the path or population. A polygonal path consists of a finite number of line segments (links) joining a sequence of points (vertices or bends). Two fundamental parameters to consider are the cost of path, $c(P)$ and the distance between path and population, $d(P, S)$. The cost $c(P)$ can be the length or number of vertices (or links) of the path, and the distance between the path and population, $d(P, S)$ can be expressed as $d(P, S) = \min\ d(p_i, P)$ or $d(P, S) = \max\ d(p_i, P)$ in *minmax* or *maxmin* problems, respectively. Distance between a point p_i and a path P is given by $d(p_i, P) = \inf_{x \in P} d(p_i, x)$ where $d(\cdot)$ is the distance between points given by a metric, generally the Euclidean distance.

Because the cost of a path is given by length notion, an efficient approach for computing shortest path will be useful in many optimization problems. For this problem, Dijkstra's approach is as follows (Cormen et al., 2001). Let $G = (V, E)$ be a directed graph with all nonnegative edge weights, the goal is then to find a shortest path from a source s to all vertices in the graph. Let $\delta(u, v)$ denote the shortest path distance from vertex u to v. For every vertex v, a distance label $d(v) = \delta(s, v)$, a parent $p(v)$ and a status $S(v) = unreached$, *labeled* or *scanned* are maintained. Initially $d(v) = \infty, p(v) = nil$, and $S(v) = unreached$ for all vertices. The method starts by setting $d(s) = 0$ and $S(s) = labeled$. While there are labeled vertices, the method picks a labeled vertex v, scans all arcs out of v, and sets $S(v) = scanned$. For scanning an arc (v, v'), one checks if $d(v') > d(v) + weight(v, v')$, and if true, sets $d(v') = d(v) + weight(v, v')$, $p(v') = v$, and $S(v') = labeled$. Since the weights are nonnegative, the graph has no negative cycles, and the method terminates with correct shortest path distances and a shortest path tree defined by parent pointers. Implementation of Dijkstra's algorithm requires time $O(V^2)$ or $O(E \log V)$ where V denotes number of vertices and E number of edges. Raman (1997) gives a survey for more efficient approaches to compute all shortest paths from a single source in graphs with nonnegative edge weights.

The objective functions considered here reflects the interaction (distance) between path and population. Sometimes the path can be considered *pulls* in the sense that it is desirable to have interacting population as close as possible. In this case, a suitable criterium is the *minmax*, that is, to minimize the maximal distance between path and population. This is called a *covering path* to a solution for a minmax problem with following formulation:

Minmax problem: Given a cost k, find a path P minimizing the maximum distance t demand points among those with cost $c(P) \leq k$.

On the contrary, other interactions are *push* meaning that the general goal is improved when the interaction (distance) increased. Then, an *obnoxious path* should be searched, and the criterium becomes the so-called *maxmin* to maximize the minimum distance between path and demand points. The o*bnoxious path problem* can be formalized as follows:

Maxmin problem: Given a cost k, find a path P maximizing the minimum distance to the demand points among those with cost $c(P) \leq k$.

Since the continuous path design has a geometric nature, the problems here are focused from the computational geometry point of view. The central aim in computational geometry is to develop data-structures and techniques to solve the problems efficiently. de Berg et al. (2000) is suggested for an introduction and the

Web sites (http://erikdemaine.org/, http://compgeom.cs.uiuc.edu/ jeffe//compgeom/ compgeom.html) for general information on publications, software, open problems and research on this issue. Efficiency is measured by computational effort and expressed with *big O* notation for time and space requirements: if n is the measure of quantity of data required to state the problem, an algorithm has complexity $O(f(n))$ if the number of elementary operations it involves is bounded by Cn where C is a constant and independent of n. Many optimization problems are nondeterministic polynomial-time (NP)-complete ones (their solutions can be quickly checked for correctness), and an *exact solution* is not possible. However, near-optimal solutions are often good enough in practice. For this, approximation algorithms return near optimal solutions. An a*pproximation algorithm* has a *ratio* bound $r(n)$ ($r(n)$ can be a constant) if, for any input size n, the solution produced by an approximation algorithm is within a factor of $r(n)$ of an optimal solution. An *approximation scheme* is an algorithm that takes the instance of the problem and a value of $\varepsilon > 0$ as inputs. For any fixed ε, the scheme is an approximation algorithm with *relative error bound* ε. Approximation schemes that run in polynomial time with size n of the input instance, the approach is called *polynomial-time approximation scheme* (*PTAS*) (Cormen et al., 2001).

23.3 COVERING SHORT PATHS

A generic application of covering short path problem is the design of a bi-level transportation structure where the path corresponds to a primary vehicle route, and all points not on that route are within easy reach (Figure 23.2). For example, the problem may be to locate postboxes (vertices of the path) on the domain in such a way that all users are located within reasonable distance from a postbox, and that the cost of a collection route through all postboxes is minimized. The problem has a *continuous version* where the vertices of the path (traveling stops) are located in a continuous region or a *discrete version* with selected vertices from a discrete set of points. As a discrete model, the *Traveling Circus Problem* (Revelle and Laporte, 1993) can be given where the problem is for the stops at a number of locations during a season to be always accessible by unvisited populations. In this context, two goals are considered: the cost of the path (length, number of vertices or bends) and the cost

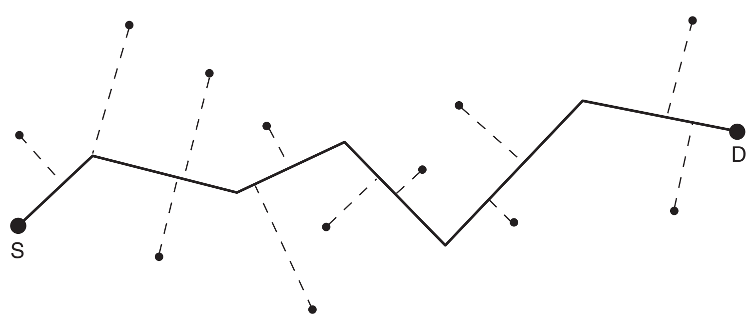

FIGURE 23.2 Bi-level transportation system.

of coverage (maximum distance to the population). These cost functions can be considered as objective function or constraints to the following problems.

23.3.1 Minimizing the Cost (Length-Vertices) of the Path

In this model, the goal is to determine a minimum length (vertices) path such that every demand point lies within a fixed distance $\varepsilon \geq 0$ from the path. In general, this problem is NP-hard (the hardest problems in the nondeterministic polynomial-time) as it reduces to a *Traveling Salesman Problem* (*TSP*) when the distance from the demand points to the route is 0. This is called a *direct covering* if the path is Hamiltonian, that is, the path passes through all points. An *indirect* covering, on the other hand, means the maximum distance between the path and the demand points is given by ε.

Direct covering: Direct covering problems are related to the TSP problem and its variants. This is a classical problem in combinatorial optimization and extensively studied even in continuous domains (Lawler et al., 1985). This problem is NP-hard even for points in the plane (Garey et al., 1976; Papadimitriou, 1977). Many heuristic and approximation algorithms have been proposed for direct covering problems (Arora, 1998). In the *quota-driven TSP*, each point to visit has an associated integral value, and the salesman has a given integer quota. The objective is, for this case, to find the shortest path with sum of the values for visited points (Awerbuch et al., AQ3
1995). The *k-TSP* takes an integer k as input and requires that the covering shortest path visits some subset of k demand points (Garg, 1996). Some recent studies considered the *Max TSP* where the objective is to maximize the length of the covering path. In Fekete (1998), the proof on the NP-hardness of the *Max TSP* is given for Euclidean spaces of three dimensions or higher. However, the complexity still remains for two dimensions with the Euclidean distance. A polynomial-time algorithm can be done in metrics defined by a convex polytope in R^d for some fixed d (Barvinok et al., 1998). For instance, the algorithm requires $O(n^2 \log n)$ time for the AQ4
L_1- and L_∞-metrics. In fact, the main idea here is to solve the problem by using a reduction to a transportation problem with a bounded number of customer locations in an appropriate bipartite graph. The *TSP with neighborhoods* is a natural generalization of the Euclidean TSP: given a set of k objects in the plane, called neighborhoods, find a shortest path that visits at least one point in each set of k *neighborhoods*. The neighborhoods can be connected (disks, polygonal regions, etc.) or disconnected sets (set of discrete points, set of polygons, . . .). Since this problem is a generalization of the *TSP*, it is by nature, NP-hard. The problem was first studied by Arkin and Hassin (1994) where they proposed $O(1)$-approximation algorithms for some kinds of neighborhoods with running time $O(n + k\log k)$ with n, total complexity of the k objects. Mata and Mitchell (1995) provided a general framework giving an $O(\log k)$ approximation algorithm with time complexity $O(n^5)$. This result was later improved to $O(n^2 \log n)$-time complexity (Gudmundsson and Levcopoulos, 1999) where a *polynomial-time approximation scheme* (*PTAS*) was given for the special case when the tour was "short" compared to the size of the neighborhoods. Recently, a polynomial time method that guaranties a constant factor approximation was devised by de Berg et al. (2005) for disjoint convex set of

arbitrary size. Furthermore, in this study, it was proved that a *PTAS* did not exist for this problem unless $P = \text{NP}$ in contrast to the standard *Euclidean TSP* for which there is a *PTAS* allowing one to compute for any given $\varepsilon > 0$, a $(1+\varepsilon)$-approximation in $O\left(n^{O\left(\frac{1}{\varepsilon}\right)}\right)$ time (Arora, 1998).

A related problem is the so-called *Errand Scheduling Problem* (Slavik, 1997) with a given collection set of points, and the aim is to visit at least one point from each set. The difference from the *TSP with Neighborhoods* is that each set to be visited has a finite number of points. An interesting version of this problem is the *Trip Planning Problem* (*TPP*) (Li and Cheng, 2004): Given a set of n points in the plane and k colors where k is a fixed constant, $k \leq n$ such that each point is associated with one color, a given starting point s and a destination point d, find the shortest trip that starts at s, passes through at least one point from each color and ends at d. This problem is a generalization of the *TSP*, and it is NP-hard. Study of heterochromatic minimum substructures in a graph is also related to this topic (Brualdi and Hollingsworth, 2001; Suzuki, 2006; Li and Zhang, 2007). The *TPP* has its origins in many applications such as: *route location* (someone is traveling for one place in town to another visiting a bank, a supermarket, a newspaper stand, etc., on the way); *advanced internet systems* (maps of navigation, planning of tasks in Google, etc.) and *computer networks* (a computer network and a set of jobs such that each job should be executed only by a specific set of nodes in the network, then the objective is to find the shortest path that visits one node in each category).

In a recent study by Díaz-Báñez et al. (2007), it was proved that the problem can be polynomially solved under some restrictions on the path with monotony and a prefixed order of the colors to visit. A polygonal chain is *ordered* if the visits are in different categories or colors according to the fixed order. Let $P = (p_1, p_2, \ldots, p_k)$ be the polygonal chain with vertices $p_1, p_2, \ldots, p_k$ ordered by $(1, 2, \ldots, k)$. Let $\theta \in (0, \pi)$ be an angle that determines certain orientation. A polygonal chain P is *monotone in this orientation* if every line with orientation $0 + \frac{\pi}{2}$ intersects P in a connected set. Similarly, it can be said that P is *monotone respect to line l* if the projections of the points $p_1, p_2, \ldots, p_k$ on l are according to the order $(1, 2, \ldots, k)$. The polygonal chain C is *monotone* if there exists an orientation θ (or a line $l(\theta)$) (Figure 23.3). The property of monotony has been widely used in modeling transportation and robotic problems since somehow going back increases the costs to simplify the structure to be determined (Arkin et al., 1989; Díaz-Báñez et al., 2000). The proposed algorithms

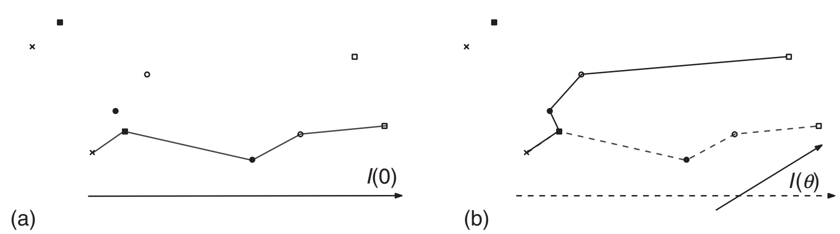

FIGURE 23.3 (a) Solution for monotony in the x-direction and order (cross, square, disk, circle, box), (b) solution for monotony in other direction.

in Díaz-Báñez et al. (2007) run in time $O(n\log^2 n)$ and $O(n^3\log n)$ when the direction of monotony is fixed or free, respectively.

The *length-constraint orienteering problem* has been considered in the operational research field: Given a set points S in the plane, a starting point s and a length constraint l_0, one needs to find a tour starting at s that visits as many points of S as possible and of length not exceeding l_0. Some related problems include the *prize-collecting traveling salesman problem* and the *vehicle routing problem*. They arise from real world applications such as delivering goods to locations or assigning technicians to maintenance service jobs. A substantial amount of work on heuristics for these problems can be found in the operations research literature (Toth and Vigo, 2002; Arora, 2003). More recently, Chen and Har-Peled (2006) presented a $(1-\varepsilon)$-approximation algorithm for the length-constraint problem that runs in $\left(n^{O\left(\frac{1}{\varepsilon}\right)}\right)$ time and visits at least $(1-\varepsilon)k^*$ points of S where k^* is the number of points visited by the optimal solution. The algorithm was also reported to work in higher dimensions.

Indirect covering: Indirect covering path is a path that is not required to pass through the demand points. Some problems for indirect covering path have been studied both for discrete and continuous demand.

In the *distance-constraint orienteering problem* or *bank robber problem*, the salesman is allowed to travel at most a distance from the points to cover, and the goal is to maximize the number of visited points subject to distance restrictions. Since it is a generalization of TSP, the problem is clearly NP-hard. The first result for geometric instances was given by Arkin et al. (1998) where a 2-approximation algorithm was obtained.

For a continuous demand i.e., the set of point to cover is not discrete, but a continuous region mentioned here as a collection of problems was well studied in the computational geometry. In the *lawn mowing and milling problems*, the covering is given by motion of a disk of a fixed radius ε in a region. The "cutter" can cross the region boundary (lawn mowing) or stay within the region (milling). Both problems are NP-hard and efficient constant-factor approximation algorithms for both problems were given by Arkin and Fekete (2000). In the transportation area, this problem can be viewed as a irrigation system. Other issues for this problem were discussed by Held (1991). AQ5

Although the problem arises naturally in the area of automatic tool path generation for pocket machining, it can be considered a generalization of the geometric *TSP* with "*mobile client*": Find the shortest path for a salesman visiting a given set of clients, each of which is willing to travel up to distance in order to meet the salesman. Arkin and Hassin (1994) gives a discrete version of this problem where a discrete set of points must be "mowed." Another problem related to a particular indirect covering is the *watchman route problem*: Find a shortest possible path within a polygonal region R such that every point in R is seen by some point in the path. The Euclidean version of the watchman route has been extensively studied (Chin and Ntafos, 1991, 1998; Carlsson et al., 1999; Tan et al., 1999), and it is NP-hard (clearly, from Euclidean TSP). Recently, Arkin et al. (2003) studied the *minimum-link watchman tour problem* where the objective is to minimize the number of links in a polygonal route rather than its length. Another related problem is the *Zookeeper's problem* where the goal is to find a shortest cycle in a simple polygon P (the zoo) through a given vertex (the zookeeper's chair) (Chin and Ntafos, 1992; Jonsson, 2003) such that the cycle visits every set of k disjoint convex polygons (cages), each sharing an edge with P without entering any of the cages. This problem is a special case of the *TSP* with neighborhoods within a simple polygon. Finally, the *Aquarium keeper's problem* by Czyzowicz (1991) where the cycle (path) touches every edge of P is presented as an indirect covering problem.

23.3.2 Minmax Covering Problems

The minmax problem, to compute a path P such that the maximum distance to the population is minimized, has as trivial solution for the path passing through all demand points. Thus, some constrained problems are considered to control the cost for building the route. With this criterion, the usual constraints on the route to be constructed arise from two factors. First, number of bends (vertices or corners) of the polygonal path plays an important role for transportation of heavy vehicles or when there is no room to maneuver. On the other hand, in a more general routing scenario, polygonal route length could be more important than the number of vertices. The *Bend (Length)-Constrained Minmax Problem* can be formulated as follows:

$$\min_{P} \max_{p_m \in S} (p_m, P) \quad \text{s.t.} \quad v(p) \leq k(l(P) \leq l_0)$$

A great number of such problems have been solved from computational geometry viewpoint. In fact, a problem closely related to the search of bend-constrained minmax polygonal routes is the approximation of polygonal curves. In various situations and applications, images of a scene have to be represented at different resolutions. A topic studied in computational geometry and applied to approximate boundaries of complicated figures in cartography, pattern recognition and graphic design (Chin et al., 1992) is to approximate the piecewise linear curves by more simple ones. Among the research in this field (Guibas, 1993; Chan and Chin, 1996), AQ6 the problem of approximating a given polygonal curve by another has been studied. In these studies, the vertices of the new curve were assumed to either have the same abscissas as given vertices in S, or they consisted of a subset of the vertex set of the original polygonal curve. Hence, if the problems for the transportation area were planned, a route with stops either on points with the same abscissas as the demand points or on the given population points should be searched. A general strategy was to divide the problem into sub-problems and use one as a subroutine for the other:

Min-ε problem: Given $\varepsilon \geq 0$, find a polygonal route P with minimum number of vertices such that $d(p, S)$ is not greater than ε.

Min-# problem: Given k, find a polygonal path P minimizing the distance $d(p, S)$ among those with a number of vertices not greater than k.

This type of problems admits several variants that arise while imposing constraints on location of the vertices or considering various types of distance. In order to remain focused on transportation, two approximation distances are considered,

$d_2(P, S)\max_{pm \in S}$ $d_2(p_m, P)$ and $d_v(P, S)\max_{pm \in S}$ $d_v(p_m, P)$ when $d_2(p_m, P)$ and $d_v(p_m, P)$ are the distance from the point to polygonal path induced by Euclidean and vertical distances, respectively. From the facility location point of view, the Min-ε problem with d_2 and d_v distances becomes an *Euclidean minmax* and a *Fitting min-max problem*, respectively. The idea of using the methods of polygonal approximation area to calculate an optimal route was first proposed by Díaz-Báñez (1998).

The approach for solving the *Vertices-Constrained Minmax Problem* was first presented by Imai and Iri (1986) as follows: first generate a set Γ of candidate distance values in such a way that the polygonal path P* searched for one of the values as distance $d(P, S)$. To each candidate value δ in the set, minimal number of vertices of a polygonal curve $P(\delta)$ that can be constructed with error at most δ (the solution of a Min-# problem) can be associated. Finally, the smallest $\delta^* \in \Gamma$ where associated length is not greater than k searched to get $P^* = P(\delta^*)$.

The most efficient algorithms for Euclidean distance problems were devised by Chan and Chin (1996). They give an $O(n^2)$ and an $O(n^2 \log n)$-time complexity algorithm for Min-# and Min- ε problem, respectively. They further showed the solution of two problems in $O(n)$ time if the points representing the population form a part of a convex polygon. Note that the Euclidean case was only solved when the vertices or bends of the new polygonal path were a subset of the original set of points. This is the called *discrete k-bend polygonal route*.

On the other hand, the problem with respect to vertical distances appears in several and important disciplines as statistics, computer graphics or artificial intelligence. The k-bend constrained minmax problem with vertical distance was first posed by Hakimi and Schmeichel (1991) where they solved two variants of the problem: when P was required to have its vertices on points in S, the *discrete problem*, and when its bends can be on any point in the plane, it is called *free problem*. For both, they developed $O(n^2 \log n)$ time algorithms which did not work in the presence of degeneracies, i.e., they did not admit points with the same x-coordinate. The approach was similar to the general method used in Imai and Iri (1986). With respect to the free problem by Wang et al. (1993), it was shown how to use a clever plane sweep procedure to find a best k-bend approximation under the Chebychev measure of error in $O(n^2)$ time. More recently, several applications of the parametric searching technique were proposed by Goodrich (1995) to solve the problem for any such k in $O(n \log n)$ time. However, in the context of facility location and transportation, the points of S (potential users) were to be determined in any position. For this reason, a dynamic programming procedure was applied in Díaz-Báñez (1998) to remove non-degeneracy assumption in the discrete case. Besides, a nice observation made here was that the algorithm to solve the discrete k-bend constrained minmax problem could be adapted to find the length constrained minmax problem.

Until now, minmax problems where a general polygonal chain should be determined were reviewed. Next, some optimization problems for polygonal chains with a particular configuration will be mentioned.

Rectilinear path: A *rectilinear path* is a chain of consecutive orthogonal segments (links) such that the extreme segments are in fact half-lines with the same slope. A rectilinear path is monotone with respect to a given orientation α if every line with

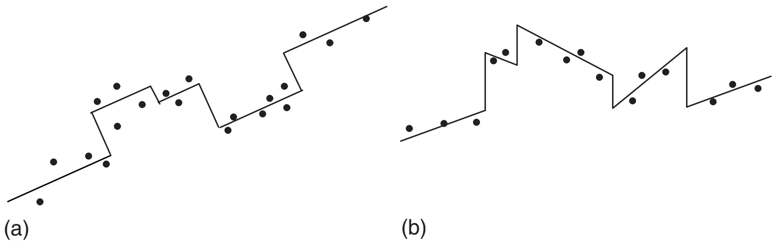

FIGURE 23.4 (a) Monotone rectilinear chain, and (b) monotone non-rectilinear chain.

slope tan $\alpha + \frac{\pi}{2}$ intersects the path either in a point or in a segment with slope tan $\alpha + \frac{\pi}{2}$ (Figure 23.4). The location of this special type of a polygonal chain was considered by Díaz-Báñez (1998). Generally, this kind of path appears in problems involving transportation routing design with applications such as floor planning, manufacturing environment design, robot moving, etc. In Díaz-Báñez and Mesa (2001), the minmax location of a monotone rectilinear route R was studied. The constraints were either the number of vertices $v(R)$ or the length $l(R)$, and the vertical distance was considered: $\min_R \max_{p_i \in S} d_v(p_i, R)$ s.t. $v(R) \leq (l(R) \leq l_0)$. This problem was first solved in $O(n^2 \log n)$ time by Díaz-Báñez and Mesa (2001). This result was further improved to $O(n^2)$ by Wang (2002) and to min $\{n^2, nk \log n\}$ by López and Mayster (2006) where k was the number of line segments (links) forming the chain. Various approximation schemes were also presented in López and Mayster (2006).

2-links polygonal paths: This is a particular case of a very simple polygonal path. Suppose the whole population is required to split into two groups such that every group uses a part of the path. Thus, the path is reduced to a chain consisting of two links. The so-called *double-ray center problem* is defined as follows. Given the population set (n points S in the plane), a configuration needs to be found, $C = (O, r_1, r_2)$ consisting of a point O in the plane and two rays, r_1, r_2 emanating from O such that the Hausdorf distance from S to C is minimized (Figure 23.5). The Hausdorf distance from

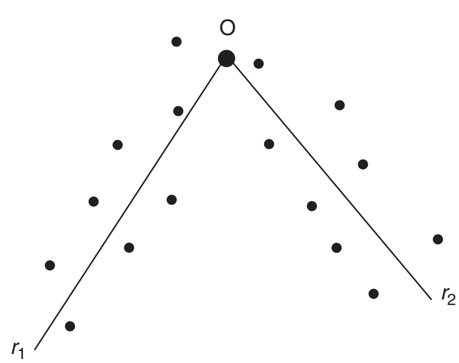

FIGURE 23.5 Double-ray with Euclidean distance.

S to C is defined y: $h(S, C) = \max_{p \in S} \min[d_2(p, r_1), d_2(p, r_2)]$ where $d_2(p, r)$ denotes the Euclidean distance between the point p and the ray r. The distance between p and r starting at point O is defined by the distance between p and the line l through r if the perpendicular line to l through p intersects r and the distance between O and p, otherwise. This problem was solved in time $O(n^3\alpha(n) \log^2 n)$ by Glozman et al. (1999) where $\alpha(n)$ was the inverse Ackermann function (Sharir and Agarwal, 1995). The authors applied the *parametric search technique* (Megiddo, 1983), a popular tool in the computational geometry field. The reader is refereed to Goodman and O'Rourke (1997) for a comprehensive survey of this technique.

On the other hand, efficient algorithms for finding *2-links* (*1-bend*) *polygonal chains* were developed when the vertical distance was used instead of the Euclidean distance. In the study by Díaz-Báñez et al. (2000), the restriction that the chain must start and end at specified anchor points a and b was considered, and it was shown that the algorithms can be extended to deal with non-anchored polygonal paths. They solved two variants of the problem: when C was required to have its corner on points in S, the *discrete problem* (Figure 23.6), and when the corner can be on any point in the plane, the *free problem*. An efficient $O(n \log n)$ time algorithm for both the *1-bend discrete* and *free minmax problem* were proposed with any of them making degeneracy assumptions. Besides, the *1-bend discrete* case can be solved within the same time bound as the problem in which a and b are not fixed but both satisfies a feasible set of linear constraints. Such problems lead to algorithms of quadratic complexity as shown by Díaz-Báñez (1998). However, by using more structure and suitable incremental updating, more efficient solutions are possible. Their procedures help determine the solutions of *1-bend polygonal chain problems* with vertical distance in $O(n \log n)$ time providing $O(n \log^2 n)$ versions of the algorithms using more elementary approaches and become easy to implement.

23.4 OBNOXIOUS PATHS PROBLEMS

Management of hazardous material (*hazmat*) is an extremely complex issue environmental, engineering, economic, social and political concerns. There is a general agreement that shipment of these type of materials is sizable and growing. Thus, development of planning criteria for minimization of industrial risks requires application of efficient techniques. In all processes transforming raw materials into final products, by-products are also generated. Some of the by-products are dangerous and have to be removed to special locations to process. An immediate problem one may

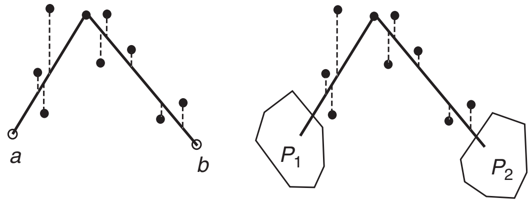

FIGURE 23.6 One-discrete problem with vertical distance.

think of is that of finding routes as far as possible from the surrounding cities. Also, in food industries, transportation of dangerous products arises, for instance, in the management of chemical substances required in the production. Large quantities of hazmats are shipped on trucks and consequences of accidents are severe (Abkowitz and Cheng, 1998). Authorities at different levels take measures to mitigate the risks associated with transportation of hazmats (U.S. Department of Transportation, 1998).

Minimization of risks for such problems has been extensively studied in a space network. CCPS (1995) gives an excellent resource for hazmat risk assessment. Batta and Chiu (1988); Boffey and Karkazis (1995); Erkut and Newmann (1995); and Erkut et al. (2005), on the other hand, present operational research studies for hazmat transportation in space networks. However, little progress has been reported in continuous spaces. Since accidents involving hazardous materials may occur during transportation, continuous context must be taken into account. In some situations, a risk reduction could pass through a re-definition of the transport system, and new better paths should be constructed. First of all, in a continuous space it is necessary to consider a restriction on the path, otherwise the path to the infinity can be removed. Two problems arise when considering constraints either on spacial situation or length of the polygonal curve:

- *Region-constrained problem*: given a polygonal region R containing the point set S to find a polygonal path P within R maximizing $D(S,P)$, i.e., $\max_{p \subset R} \min_{p_m \in S} d(p_m, P)$.
- *Length-constrained problem*: given a positive value l_0, to find a polygonal path P with length-bound l_0 maximizing $d(S; P)$, i.e., $\max_{p:l(P) \leq l_0} \min_{p_m \in S} d(p_m, P)$.

A related problem with former version was posed in Drezner and Wesolowsky (1989). This problem showed, in a given a polygonal region R, containing point set S, computing a polygonal path within P connecting two points a and b (or two segments of R) such that the minimum distance to S is maximal. The authors provided an approximate algorithm for calculating such an anchored obnoxious route. However, this problem could be easily solved by using the Voronoi diagram, $V(S)$ of the point in S. Aurenhammer (1991) gave a comprehensive survey on Voronoi Diagrams. A simple exact approach that solves this problem was included in this chapter. Since the goal is achieved if the segment giving the minimum distance maintains the same distance to either point, an optimal route walking along the edges of $V(S)$ can be found. Then, optimization problem immediately is reduced to a discrete graph problem: After labeling each edge of $V(S)$ with its minimum distance to its two sites and adding the end points a and b as new vertices to $V(S)$, a *breadth first search* from a will find a polygonal route to b in $V(S)$ where the minimum label will be a maximum within $O(n)$ time (Cormen et al., 2001). As a consequence, this problem can be solved exactly with a simple exact $O(n \log n)$-time algorithm.

An approximate algorithm for the second version, the *length-constrained problem* was proposed recently in Díaz-Báñez et al. (2005). By using several geometric techniques as Voronoi diagrams in the Laguerre geometry and shortest paths among obstacles, they have proposed a $[O(n)+O(\log(\varepsilon^{-1}))][O(n)+O(kn)]$-time algorithm where was the allowed error. This approach can be adapted to many situations with the idea to use a bisection method for the following decision problem: Place a circle of radius r at each site s_i, $i=1,\ldots,n$ (Figure 23.7) and compute the shortest path avoiding the circles. The desired path must go among the union of those circles if r is equal to the maximum of minimum distances. Next, compute the union of the arrangements of circles, and compute the contour of the union from that. Such union will be formed by many connected components (the number of components ranges from 1 to n), and the one that contains a and b should then be found. Finally, the shortest path P between a and b among the circular obstacles can be obtained. If $l(P)$ is equal to the bound l_0, then a solution of the problem where r is the maximum achieved is computed. These techniques can be useful when demand population is modeled by polygonal regions as well. Also, the problem can be addressed in a similar way for other distances, for example, induced by polyhedral metrics. In the particular case of the L_∞- or L_1-metrics, efficient algorithms can be found (Díaz-Báñez et al., 2005).

2-links polygonal paths: As seen above, placement of an obnoxious path can be modeled by a polygonal chain with one corner (two links). Apart from a spacial constraint (as otherwise the route may be simply removed to infinity), a constraint on the length of the chain was considered in Díaz-Báñez and Hurtado (2006). They also added the restriction that the chain had to start and end at specified anchor points a and b corresponding to given origin and destination. The so-called *maximin 1-corner polygonal chain problem* is stated as follows: Given a set S of points in the plane and a positive value l_0, find a 1-corner polygonal route, P with Euclidean length $l(P)\leq l_0$, such that $\min_{p_i\in S} d(p_i,P)$ is maximized among all possible chains fulfilling the conditions.

Due the geometric nature of the problem, the resolution is addressed from the computational geometry point of view. A *boomerang* is the area swept by a disk with its center describing the 1-corner route. Thus, in a geometric setting, the problem asks to find the *largest empty boomerang* anchored at a and b. Let us observe that the restriction on the length implies that the vertex cannot be exterior to an ellipse with focus at a and b. This defines a continuous search space; however, a discrete set of candidate placements can be generated (Figure 23.8). The study by Díaz-Báñez and

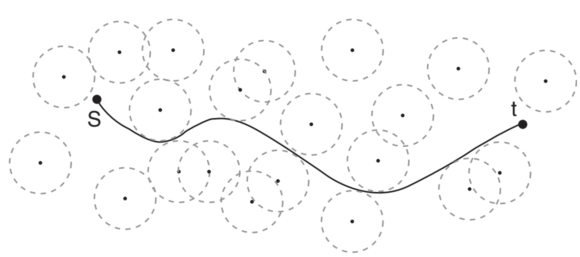

FIGURE 23.7 Finding the shortest path among circles.

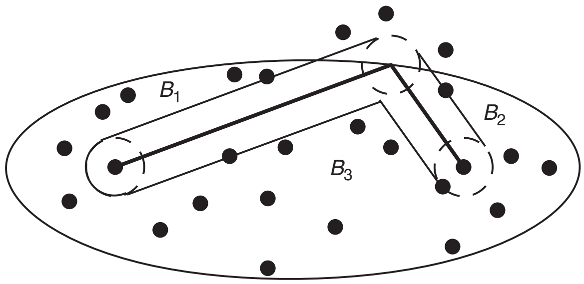

FIGURE 23.8 Empty anchored boomerang and its three parts.

Hurtado (2006) proposes an $O(n \log n)$-time algorithm for finding a 1-corner obnoxious polygonal chain whose length is exactly l_0, and an $O(n^2)$-time algorithm when the length of the optimal chain is at most the given bound l_0. The placement of empty geometric objects (circle, rectangle, unbounded rectangular strip, annulus, etc.) of "maximum measure" among a set points have been extensively studied (Cheng, 1996; Houle and Maciel, 1998; Mukhopadhyay and Rao, 2003; Nandy and Bhattacharya, 2003; and Díaz-Báñez et al., 2003). Motivation for computing optimal empty figures comes from a variety of practical problems such as robot manipulation (Houle and Maciel, 1998), computer-aided design (Nandy and Bhattacharya, 2003) and collision-free routing for transport objects through a set of obstacles (points) (Cheng, 1996). It is of particular interest to use empty corridors (Figure 23.9). An L-*shaped corridor* is the concatenation of two perpendicular links (a link is composed by two parallel rays and one line segment forming an unbounded trapezoid). The *angle* of a L-*shaped corridor C* is the angle determined by their rays. A widest empty L-*shaped corridor* can be calculated in $O(n^3)$ time and $O(n^3)$ space (Cheng, 1996) when the interior angle is fixed: $\alpha(C) = \frac{\pi}{2}$. More recently, Díaz-Báñez et al. (2006) relaxed the angle constraint and allowed α (C) to assume arbitrary values. The proposed algorithm computed a widest empty 1-corner corridor in $O(n^4 \log n)$ time and $O(n)$ space.

Another kind of empty corridor, called *siphon*, was recently addressed in Bereg et al. (2007). A *siphon* was the locus of points in the plane that were at distance w

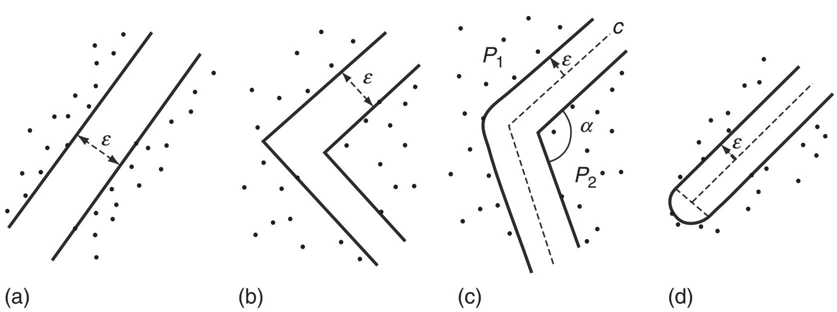

FIGURE 23.9 (a) Corridor, (b) L-corridor, (c) α-siphon, (d) silo.

from 1-corner polygonal path P where w is called the *siphon width*. Notice that a siphon is the area swept by a disk with its center describing the 1-corner polygonal chain. An α-siphon is a siphon such that the interior angle α of P, called the *siphon angle*, is fixed. The *widest empty siphon problem* can be stated as follows: Given a set S of n points in the plane and a fixed value α, $0 \leq \alpha \leq \pi$, compute the α-siphon with the largest width w such that no points of S lies in its interior.

The widest α-siphon gives a "better" solution than the L-shaped corridor of Cheng in the following sense: Suppose that transporting a circular object (a disk) with radius w through a set P of obstacles (points) is of interest. Then, the decision problem becomes: *Is there a 1-corner polygonal path for transporting the disk through P without collision*? Notice that the non-collision property means disk center (vehicle for transportation) pass at distance at least w from the points (population). The algorithm for L-shaped corridor can produce a negative answer while the α-siphon's method gives an affirmative answer since the width of the widest α-siphon is always larger than or equal to the width of the widest L-shaped corridor. In fact, every L-shaped corridor contains a siphon of the same width, and the reciprocal is false (Figure 23.10).

In Bereg et al. (2007), three variants of the widest α-siphon problem were considered: (1) the *widest oriented α-siphon problem* where the angle α and orientation of one of the half-lines of 1-corner polygonal path P was known; (2) the *widest arbitrarily-oriented α-siphon problem* where only the angle α was known; and (3) the *widest anchored and arbitrarily-oriented α-siphon problem* where the corner of P was anchored at a given point. An efficient $O(n \log^3 n)$-time algorithm for the first problem was given, and the arbitrarily oriented case was solved with an $O(n^3 \log^2 n)$-time algorithm. Finally, the problem of computing the widest

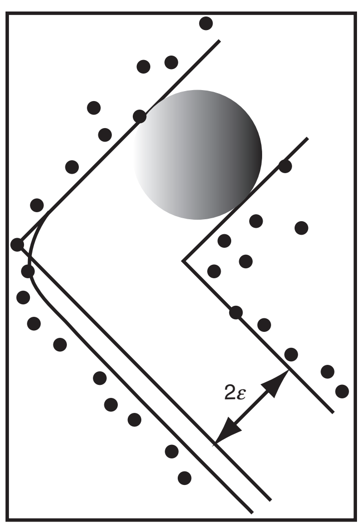

FIGURE 23.10 Width of the widest α-siphon is larger or equal than the width of the widest L-corridor.

anchored and arbitrarily-oriented siphon were solved within an optimal $O(n \log n)$-time algorithm.

23.5 OTHER OPTIMAL PATHS

23.5.1 Multiple Objective Problems

Due to the inherent multiobjective nature of many routing problems, there has been a tremendous increase in multiobjective optimization problems in networks (Figueira et al., 2005). For an interesting routing problem, see the *minimum-covering/shortest-path* in Current et al. (1994) where a two-objective path problem was addressed: minimization of total population was negatively impacted by the path and minimization of the total path length.

Typically, many models are designed primarily to generate *Pareto-optimal* solutions. A path is called *Pareto-optimal* or *efficient* if no other path has a better value for one criterion without having a worse value for other criterion. For example, one may wish to compute a short polygonal path with few vertices. Note that a minimum-vertices path may be far from optimal with respect to length; similarly, a shortest path may have many vertices while there exists a path connecting source and destination with few links (legs). Experimental studies suggest that average number of Pareto-optimal is small in practice although this number can be exponential in theory.

In a continuous domain, criteria of interest are: length (Euclidean or other metrics), total number of vertices (or links) or distance between path and population. Multi-objective optimization problems tend to be difficult even for two criteria (Arkin et al., 1991): find a path in a polygonal domain whose length is at most l_0, and total number of vertices is at most v_0 is NP-hard; finding the shortest path constrained to have almost k links is a current open problem, and no exact solution is actually known. A comprehensible text by Ehrgott (2005) on multi-objective problems is also suggested for further reading.

23.5.2 Optimal Paths in Three Dimensions

Computing optimal paths in higher dimensional geometric spaces is difficult, thus most effort has been devoted on three-dimensional spaces. In fact, unlike shortest paths among obstacles in the plane, shortest paths in a polyhedral domain do not need to lie on a discrete graph. It was shown in Bajaj (1988) that the algebraic numbers describing optimal path lengths might be exponential. Moreover, the number of combinatorially distinct shortest paths connecting two points may be exponential in the input. This fact was used in Canny and Reif (1987) to prove the NP-hardness of the three-dimensional shortest path problem. In light of the difficulty of a general problem, special cases of shortest paths that can be efficiently solved have been addressed. For instance, paths on a *polyhedral terrain* (a connected subset of the space where the boundary consists of a union of a finite number of triangles) makes the problem to be two-dimensional, and the continuous Dijkstra's paradigm leads to $O(n^2)$-time algorithms (Chen and Chan, 1996). Har-Peleg (1998) and Mitchell and Sharir (2004) cover the approximation algorithms on polyhedral domains for polynomial-time algorithms for some special cases. For a complete review of three-dimensional shortest path problems, Mitchell (2000) is suggested.

23.5.3 Online Algorithms for Optimal Path Problems

The vast majority of published research on shortest paths algorithms deal with static environment, that is, the exact layout of the environment where the vehicle moves is known. The area of geographical positioning system (GPS) navigation proves to be a challenge for current planning engines, and dynamic route-planning systems are necessary. It subsumes several algorithmic issues from computational geometry in general and artificial intelligence search in particular such as autonomous robotics to gather and refine raw data by integrating different input sources as well as algorithms to build and query the graph and known novel search techniques to speed up the short path computations. The general goal is to find a *navigation strategy* that controls the motion while minimizing the objective function. There has been an increasing interest in dynamic management of transportation networks (Chabini, 1998). This results in a new family of shortest paths problems known as dynamic (time dependent) shortest paths problems. If the underlying space is a continuous environment, the problems are focused from the robotic applications point of view. The robot does not have a prior information about obstacles in the environment. In such cases, it has the information about their current location as well as the location of the goal, and the information about the environment is acquired on-line. Mitchell (2000) gives a complete survey on online algorithms in robotic.

Another interesting issue is the class of algorithms that solve global problems and wireless networks by means of local algorithms. A local algorithm is the one in which any node of a network has only information on nodes at distance at most k from itself, for a constant k. Given a set of points S on the plane, the unit distance graph associated to S is the one with its vertex set consisting of the elements of S, two of which are connected if they are at distance at most one (Figure 23.11). Unit distance graphs are used to model various types of wireless networks, including cellular networks, sensor networks, ad-hoc networks and others where the nodes

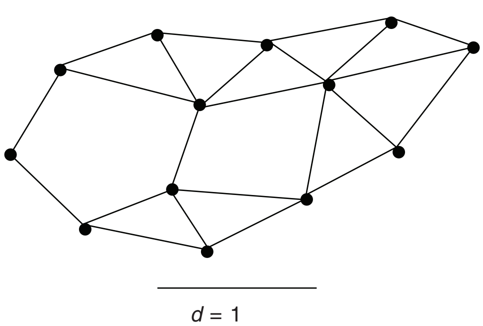

FIGURE 23.11 Unit distance graph.

represent broadcast stations with a uniform broadcast range. A recent survey on local algorithms on unit distance networks is given by Urrutia (2007). This area provides a new set of computational problems for optimal paths in a continuous domain and promises future in the research on this kind of problems since overall information is not known in advance due to the temporal changes of the environment.

23.6 CONCLUSION

In this chapter, main problems in the literature for various shortest paths problems in a continuous environment were described focusing primarily on transportation with theoretical results. There are numerous issues involved implementing the algorithms for real scenarios. Zhan (1997) gives useful information for researches and practitioners studying in network transportation area. It was concluded that, in a continuous domain, some algorithms may well be implementable and useful; however, in many cases, the implementation is too complex and may have numerous constants behind the asymptotical notation of complexity (*big O* notation). In addition, the algorithms may be not robust under uncertainties. Montemanni et al. (2004) presented a robust heuristical approach on networks. A future review should address the issues facing practitioners in the implementation of optimization algorithms for continuous shortest paths. In addition, compared to shortest paths problem for transportation on networks, the literature on continuous versions, is very limited. The geometric shortest paths issue has been mostly studied for robotic mainly when the object moves inside a polygonal region. However, there is a vast set of problems to be addressed if a well studied problem on real road networks is considered in a continuous domain. This chapter also identifies potential research areas and weak points in the existing literature in the field of transportation, and it is expected to stimulate and suggest the development of applied research in the field of transportation regarding food products.

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